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Recent cosmological observations strongly suggest that the Universe is dominated by an unknown form of energy with negative pressure. An intriguing suggestion is that this energy *tracks* the other forms of energy (e.g. matter, radiation) in the Universe, thereby explaining its closeness to the critical density today. In general tracking models fail, however, because (i) they give the wrong equation of state and (ii) they spoil the predictions of Big Bang Nucleosynthesis. We propose a class of models which track but can avoid these problems. These models require an oscillatory potential; the resultant field dynamics not only solves the missing energy problem but also leads to testable predictions in the Cosmic Microwave Background (CMB) and Large Scale Structure (LSS).

Introduction. A variety of evidence accumulated over the last several years points to the existence of an unknown, unclumped form of energy in the Universe. First was an apparent concordance [1] of different measurements: the age of the Universe; the Hubble constant; the baryon fraction in clusters; and the shape of the galactic power spectrum. Second came the stunning observations [2] of tens of distant Type Ia Supernovae, which found a distance-redshift relation in accord with a cosmological constant, but in strong disagreement with a matter dominated Universe. Finally, this past year has seen analyses [3] of the experiments measuring anisotropies in the CMB. Taken together, the CMB experiments plot out a rough shape for the power spectrum, one that is in accord with a flat Universe, but in disagreement with an open Universe. If we believe the estimates of matter density coming from observations of clusters [4], the only way to get a flat Universe, and hence account for the CMB measurements, is to have an unclumped form of energy density pervading the Universe.

Perhaps the simplest explanation of these data is that the unclumped form of energy density corresponds to a positive cosmological constant [5]. A non-zero but tiny constant vacuum energy density (cosmological constant) could conceivably be explained by some unknown string theory symmetry (that sets the vacuum energy density to zero) being broken by a small amount. However, to explain in this way a constant vacuum energy density of $2 \times 10^{-59} \, \mathrm{TeV}^4$, which is not only small but is also just the right value that it is just beginning to dominate the energy density of the Universe *now*, would require an unbelievable coincidence. A different possibility is to give up the dream of finding a mechanism which would set the vacuum energy density to exactly zero and resort to believing that anthropic considerations select amongst $\gtrsim 10^{100}$ string vacua to find one with a vacuum energy density sufficiently fine-tuned for life. Although this anthropic selection mechanism is logically consistent and even predicts a small but observable cosmological constant, one might think that nature would have found a more efficient mechanism to obtain a sufficiently small

cosmological constant than such extreme brute force application of anthropic selection.

An alternative is to assume that the true vacuum energy density is zero, and to work with the idea that the unknown, unclumped energy is due to a scalar field ϕ which has not yet reached its ground state. This idea, which is called dynamical lambda or quintessence, has received much attention [6] over the last several years. However, two problems still remain. First, the field's mass has to be extremely small, less than or of order the Hubble constant today $\sim 10^{-33} \, \mathrm{eV}$, to ensure that it is still rolling to its vacuum configuration. This is in general difficult because scalar fields tend to acquire masses greater than or of order the scale of supersymmetry breaking suppressed by at most the Planck scale: $m \gtrsim F/m_{\mathrm{Pl}} \gtrsim \mathrm{TeV}^2/m_{\mathrm{Pl}} \sim 10^{-3} \, \mathrm{eV}$. Although difficult, this could be achieved using pseudo-Nambu-Goldstone bosons [7]. Another more speculative way to achieve this would be to use the hypothetical symmetry (perhaps some sort of hidden supersymmetry) that ensures that the true vacuum energy density is zero to also protect the flat directions in scalar field space that would correspond to the very light scalar fields necessary for quintessence. The second, and perhaps even more serious problem is that almost all of these models require that we live in a special epoch today, when the quintessence is just starting to dominate the energy density of the Universe, and furthermore this specialness cannot even be justified by use of anthropic arguments.

Tracking (or attractor) models alleviate this latter difficulty. The simplest tracking model [8] is one in which the scalar field energy density is always related to the ambient energy density in the Universe: if the dominant component in the Universe is radiation, then the tracking field's energy density also falls off as a^{-4}, where a is the scale factor of the Universe. If the dominant component is matter, then the field's energy density scales as a^{-3}. This behavior arises from an exponential potential for ϕ. Since the energy density in this field is always comparable to the background density, we are not living at a special epoch: any observer in the distant past or

future would also see the tracking field's energy density. However, these tracking solutions run into two problems. First, if their energy density today truly is dominant, then it should also have been dominant at the time of Big Bang Nucleosynthesis (BBN). Constraints from observations of light element abundances preclude such an additional form of energy density at early times. Second, tracking models have the wrong equation of state. Today the tracking field behaves like matter, with zero pressure, instead of having the necessary negative pressure to accelerate the Universe.

Nonetheless, despite these difficulties, the basic idea of a tracking field is so compelling that it seems worthwhile to see if a workable model can be constructed. Here we show that a modification of an attractor potential leads to an interesting, testable class of models. These models maintain the virtue of tracking models – i.e. we do not live at a particularly special time – while at the same time being able to produce energy density today with the proper equation of state, satisfy the BBN constraints, and leading to testable features in the CMB and matter power spectra. The modification to the potential is a sinusoidal modulation, which induces the tracker field to oscillate about the ambient energy density. Thus, we call this type of energy *Tracking, Oscillating Energy*, or TOE.

Recently a number of authors [9] have generalized the notion of tracking to include models in which ϕ does not exactly track the ambient density. For example, potentials like $V = V_0\phi^{-n}$ or $V = V_0\exp(1/\phi)$, as opposed to pure exponential potentials, can, for suitable choices of V_0, catch up with the critical density late in the evolution of the Universe and thus provide a natural setting for explaining why the Universe is accelerating today but wasn't at nucleosynthesis. However, the suitable choice of V_0 must be of the order of the critical energy density today, i.e. we are back with the problem of us living at a special epoch today and not even being able to use anthropic arguments to justify this specialness.

The potential and the field evolution. Consider a scalar field ϕ with potential $V(\phi) = V_0\exp(-\lambda\phi)$ where here and throughout we use units in which $8\pi G = 1$. It is well-known [8] that such a potential leads to an attractor solution with $\Omega_\phi \equiv \rho_\phi/(\rho_\phi + \rho_o) = n/\lambda^2$ where ρ_o is the energy density in the other component of the Universe, which is assumed to scale as a^{-n}. Thus, no matter what the initial conditions are for ϕ, it always evolves so that it tracks the rest of the density in the Universe.

Now consider the potential

$$V(\phi) = V_0\exp(-\lambda\phi)\left[1 + A\sin(\nu\phi)\right]. \tag{1}$$

This potential serves to modulate the tracking behavior. Figure 1 shows the resultant evolution of ϕ and its energy density for a particular set of the parameters A, ν. Also shown is the tracking solution for this particular value of λ without the modulation. As expected, the sinusoidal term in the potential leads to oscillations about

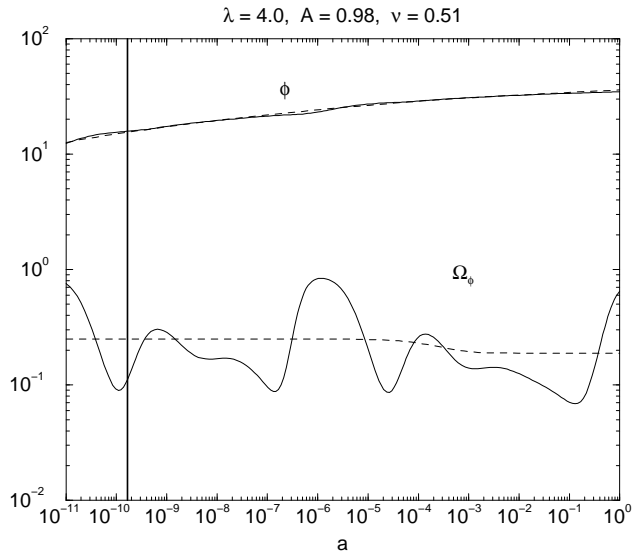this tracking behavior. The amplitude of the oscillations is governed by the parameter A and the frequency by ν.

FIG. 1. The fraction of the critical density in ϕ for the potential in Eq. (1). The dotted line shows the corresponding tracking solution ($A = 0$). The upper set of curves shows the evolution in ϕ for the TOE and the tracking models.

The energy density due to ϕ is relatively small at the time of BBN and relatively large today for the parameter set in Figure 1. This involves a bit of fine-tuning and is therefore subject to some of the same "negative advertising" we used above. We feel, though, that the fine-tuning required here is less severe than in other models; for example, results are insensitive to the choice of V_0. Nonetheless, these aesthetic considerations may well become irrelevant because observations may soon judge which model is correct [10]. For these purposes, it is important to compute the power spectra of the perturbations in a TOE model.

Power Spectra. Perturbations evolve differently in the presence of the scalar field energy density. For example, perturbations typically grow only when the Universe is matter dominated. Therefore, we expect a non-zero Ω_ϕ to lead directly to power suppression on the scales inside the horizon, with increased suppression for larger Ω_ϕ.
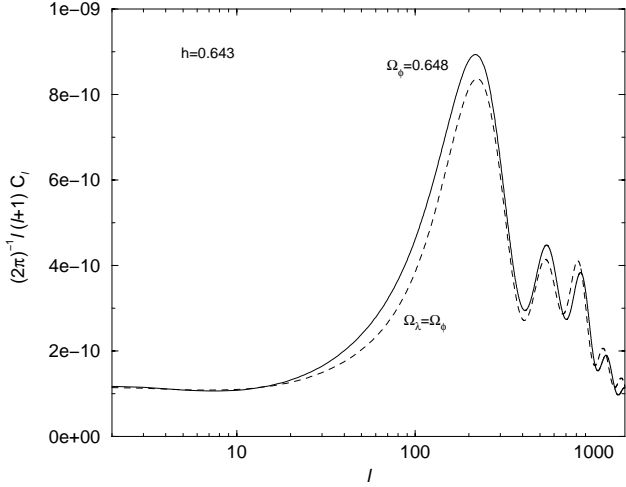
FIG. 2. The angular photon power spectrum from the TOE model of Figure 1. Also shown is a cosmological constant model with all other parameters equal.

The prediction for the CMB angular power spectrum is plotted in Figure 2. The primeval power spectrum is scale-invariant with adiabatic initial conditions. Also plotted for comparison is a model (ΛCDM) with cosmological constant $\Omega_\Lambda = \Omega_\phi$ today and the rest of the cosmological parameters also being the same. In further discussions we will contrast the results from the TOE model against this ΛCDM model. A noteworthy feature in Figure 2 is the increase in the heights of the first two peaks compared to that of the ΛCDM model. This stems from the fact that the gravitational potential decays more in the presence of the additional quintessence energy density. The decay of the potential at and after recombination (the so-called Integrated Sachs–Wolfe , or ISW, effect) leads [11] to enhanced power on scales $l \lesssim 600$, after which the potential becomes irrelevant. Note that the increase in the amplitude of both the first and second peak cannot be mimicked by adding more baryons, which raise the odd peaks but lower the even ones.

On smaller scales ($l \gtrsim 600$), the TOE model has smaller anisotropies. Here there are two competing effects. First, the difference between the TOE and the ΛCDM models (around recombination when Λ is insignificant) is the presence of the extra quintessence energy density, which leads to the expansion rate in the two models being related as–

$$H_{\mathrm{TOE}}(a) = H_{\mathrm{ACDM}}(a) \times \left(1 - \Omega_\phi(a)\right)^{-1/2} . \quad (2)$$

Eq. 2 implies that all the relevant scales at recombination (which occurs at $a_r \simeq 10^{-3}$) are smaller in the TOE model by a factor of about $\sqrt{1 - \Omega_\phi(a_r)}$. In particular, the damping scale is smaller, which increases in the power on small scales for the TOE model relative to the ΛCDM model. The second effect is the large scale

normalization of the two models*, and this second effect more than compensates for the first. COBE normalization is sensitive to scales around $\ell = 10$ for which the differences in the two models with regard to the late-ISW effect is important. In particular, since Λ domination occurs very late, the ISW contribution around $\ell = 10$ is much larger in the TOE model. This in turn implies that the normalization of the primeval power spectrum is smaller, a fact noticeable in the smaller amplitude of the photon power spectrum for the TOE model at small scales (and also the matter power spectrum, as we will soon see). One last effect that is worth pointing out concerns the difference in the peak positions in the two models (though unlike the peak amplitudes, it is probably not easily discerned). In particular, the TOE model has the acoustic features in its angular power spectrum shifted to smaller scales. This directly traces to the decrease in the angular diameter distance to the last scattering surface, for the TOE model. Of course, there is also the competing effect of the decrease in the size of the sound horizon at last scattering for the TOE model, which minimizes the effect.
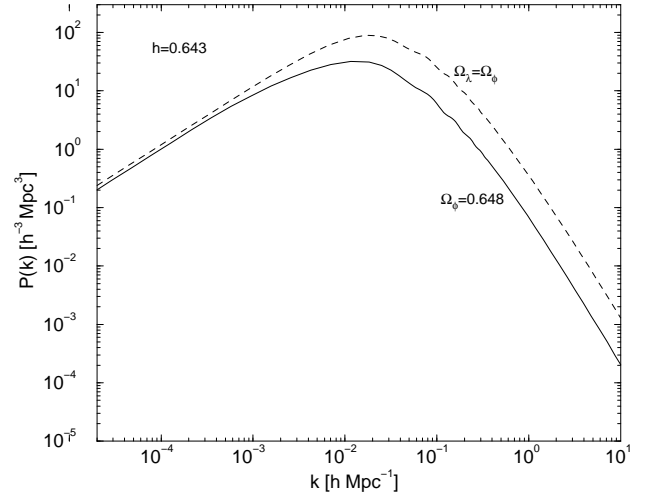

FIG. 3. The matter power spectrum from the TOE model of Figure 1. Also shown is a cosmological constant model with all other parameters equal. Power is significantly smaller in the TOE model.

The prediction for the matter power spectrum is plotted in Figure 3. The difference in power at the largest scales is due to COBE normalization and the difference in the super-horizon growth factor (which is sensitive to the equation of state of the cosmic fluid) for the perturbation. As one moves to smaller scales, which entered the horizon well before the present, the differences in the

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*Note that the normalization affects the first two peaks also: the TOE model would have an even larger relative amplitude there if not for the large scale normalization.

evolution of the matter perturbation become more pronounced. The presence of the extra quintessence energy stunts the growth of perturbation once a mode enters the horizon. So, the earlier the mode enters the horizon, the larger the growth suppression relative to the ΛCDM model. In other words, smaller modes are monotically more suppressed (something that may not be noticeable in the log plot) compared to the same modes in ΛCDM model. It might also be surprising that the ϕ domination around $a = 10^{-6}$ does not cause a more appreciable feature (*i.e.*, suppression) in the power spectrum. The reason is that the smallest scales in Figure 3 have just entered the horizon at the time of ϕ domination ($a \sim 10^{-6}$).

The normalization on the small scales is generally quoted in terms of σ_8, the rms mass fluctuation within a $8\, h^{-1}$ Mpc sphere. For the parameters in Figure 1, the TOE model has $\sigma_8 = 0.4$. This is several sigma smaller than the preferred value (see e.g. [10]) of ~ 0.8, but could be rectified by a small blue-shift in the primordial spectrum [12].

Conclusions. We have constructed a model wherein the energy density tracks the dominant component in the Universe; satisfies the BBN constraints; and has the proper equation of state today. Further, this model makes definite predictions for large scale structure and for the CMB.

Perhaps the greatest drawback of this class of models is the arbitrariness of the potential. In particular we know of no theory which predicts a potential of the form given in Eq. (1). Nonetheless, we feel that the testable predictions of the model and the aesthetic quality it preserves that we do not live in a special epoch are of sufficient interest to warrant further study.

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